

Solving For Internal Rate Of Return

Part II - The Secant Method

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In Part I we were given forward rates for an interest rate swap (variable rate leg) and were tasked with calculating the fixed rate equivalent (fixed rate leg) via the Bisection method. In Part II we will use the Secant Method to calculate the fixed rate. To that end we will work through the following hypothetical problem from Part I...

Our Hypothetical Problem

We are given the following interest rate swap parameters given that the swap has a notional value of \$1,000,000 and a term of 10 years with swap payments made at the end of each annual period.

Table 1: Interest Rate Swap Parameters

Annual Period	Forward Rate	Discount Factor	Variable Leg Payment	PV Variable Leg Payment	IRR 4.5338%
0	—	1.0000	—	—	-1,000,000.00
1	3.50%	0.9662	35,000.00	33,816.43	35,000.00
2	3.75%	0.9313	37,500.00	34,922.30	37,500.00
3	4.00%	0.8954	40,000.00	35,817.74	40,000.00
4	4.25%	0.8589	42,500.00	36,504.89	42,500.00
5	4.50%	0.8220	45,000.00	36,987.79	45,000.00
6	4.75%	0.7847	47,500.00	37,272.23	47,500.00
7	5.00%	0.7473	50,000.00	37,365.65	50,000.00
8	5.25%	0.7100	52,500.00	37,276.89	52,500.00
9	5.50%	0.6730	55,000.00	37,016.10	55,000.00
10	5.75%	0.6364	1,057,500.00	673,019.98	1,057,500.00
Total	—	—	1,462,500.00	1,000,000.00	462,500.00

Note: The IRR in the table above was derived by using the Excel IRR function.

Answer the following questions...

Question 1: Use the Secant Method to calculate the internal rate of return (i.e. the fixed rate).

Building Our Model

We will define the function $F(r)$ to be the net present value of a time series of cash flows discounted at the annual fixed interest rate r , the variable N to be the notional value of the swap, the variable C_t to be cash flow at the end of time t (variable leg payment), and the variable T to be swap term in years. The equation for the net present value of the swap at time zero is...

$$F(r) = \left[\sum_{t=1}^T C_t \left(1 + r \right)^{-t} \right] - N \quad (1)$$

The Secant method: Like the bisection method, the secant method is a root-finding method that applies to any continuous function for which one knows two values with opposite signs. The method consists of repeatedly calculating the point at which the secant line crosses the x axis and then redefining the points a and b . The secant method can be thought of as a finite-difference approximation of the Newton-Raphson method (Part III).

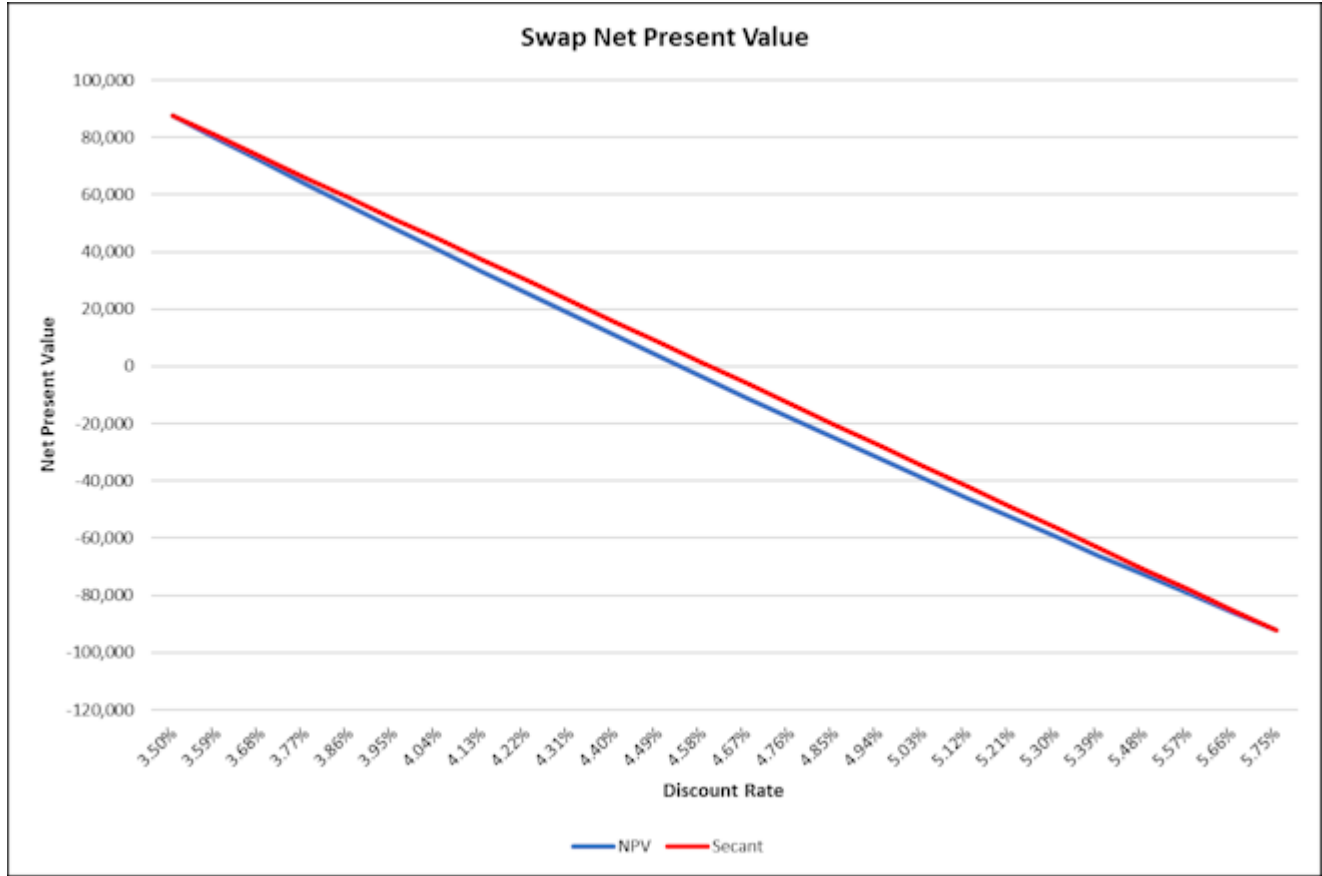
We will define the variables a and b to be interest rates where $a < b$. The definitions of these interest rate variables are...

$$\begin{aligned} a &= \text{Discount rate where } F(a) > 0 \\ b &= \text{Discount rate where } F(b) < 0 \end{aligned} \quad (2)$$

We will define the function $S(r)$ to be the secant line that connects points a and b . Using Equation (2) above the equation for the secant line is...

$$S(r) = m(r - a) + c \text{ ...where... } m = \text{slope} = \frac{F(b) - F(a)}{b - a} \text{ ...and... } c = \text{constant} = F(a) \quad (3)$$

The graph of the NPV (Equation (1)) and Secant (Equation (3)) functions for our hypothetical problem is...



Note that the Secant line crosses the x-axis at around 4.67%. Note also that the Secant line is always above the NPV line and therefore the discount rate where the Secant line crosses the x-axis will result in an NPV that is negative. Because NPV is negative we will revise the b value in Equation (2) after each iteration.

Using Equation (3) above, the equation for the discount rate where the secant line crosses the x-axis is..

$$\text{if... } S(r) = 0 \text{ ...then... } r = \frac{m a - c}{m} \quad (4)$$

To calculate the discount rate r where $F(r) = 0$, we will employ the following iteration...

Step	Action
1	Determine guess values for points a and b such that $F(a) > 0$ and $F(b) < 0$.
2	Calculate the secant equation $S(r)$ that connects points a and b .
3	Use the secant equation from Step 2 to solve for \hat{b} such that $S(\hat{b}) = 0$.
4	Use the value of \hat{b} from Step 3 above and calculate the new value of $F(\hat{b})$.
5	If $F(\hat{b}) \approx 0$ then stop, otherwise replace the value of b with \hat{b} and go to Step 2.

The Answers To Our Hypothetical Problem

Question 1: Use the Secant Method to calculate the internal rate of return (i.e. the fixed rate).

We need to determine the start and end interest rates for our iteration process. The start and end rate will be the minimum and maximum, respectively, of the forward rates in Table 1 above. We know that the minimum rate will result in a positive NPV and the maximum rate will result in a negative NPV, which is what we want. The starting rate interval will be...

$$a = 3.5000\% \text{ ...and... } b = 5.7500\% \quad (5)$$

Using the start rates in Equation (5) above, the iteration process to 6 iterations is...

Iteration	Interest Rate		NPV of Cash Flow		Secant Line		New [b] rate	
	a	b	a	b	constant	slope	Rate	NPV
0	3.5000%	5.75000%	87,672.66	-92,334.06	87,672.66	-8,000,298.53	4.5959%	-4,979.70
1	3.5000%	4.59587%	87,672.66	-4,979.70	87,672.66	-8,454,706.18	4.5370%	-251.26
2	3.5000%	4.53697%	87,672.66	-251.26	87,672.66	-8,478,936.39	4.5340%	-12.63
3	3.5000%	4.53401%	87,672.66	-12.63	87,672.66	-8,480,158.25	4.5339%	-0.64
4	3.5000%	4.53386%	87,672.66	-0.64	87,672.66	-8,480,219.68	4.5338%	-0.03
5	3.5000%	4.53385%	87,672.66	-0.03	87,672.66	-8,480,222.77	4.5338%	0.00
6	3.5000%	4.53385%	87,672.66	0.00	87,672.66	-8,480,222.93	4.5338%	0.00

Answer: At iteration 6 the discount rate converges to approximately 4.5338%.